

# IFORS2023: Comparing interactive multiobjective optimization methods with artificial decision-makers based on computational rationality

**Giovanni Misitano**<sup>1</sup>   Bekir Afsar<sup>1</sup>   Jussi Jokinen<sup>1</sup>   Kaisa Miettinen<sup>1</sup>

<sup>1</sup>University of Jyväskylä (Finland), Faculty of Information Technology

July 10, 2023



JYVÄSKYLÄN YLIOPISTO  
UNIVERSITY OF JYVÄSKYLÄ



- I am Giovanni Misitano, a doctoral researcher from the Multiobjective Optimization Group at the University of Jyväskylä (Finland).
- The topic of my thesis is *Explainable Multiobjective Optimization*.
- President of the Finnish Operations Research Society (FORS)



- 1 Motivation
- 2 Background
  - Multiobjective optimization
  - Interactive methods
- 3 A new type of artificial decision maker
  - Artificial decision makers
  - Computational rationality
  - Preferences
  - Memory
- 4 Current state of research
- 5 Conclusions

- 1 Motivation
- 2 Background
  - Multiobjective optimization
  - Interactive methods
- 3 A new type of artificial decision maker
  - Artificial decision makers
  - Computational rationality
  - Preferences
  - Memory
- 4 Current state of research
- 5 Conclusions

- Preferences provided by a decision maker play a key role.
- Decision makers and preferences are subjective.
- Success measured in ability to support decision makers.

Two important questions arise:

- ① How can we compare interactive methods?
- ② How can we support different decision makers?

We will combine ideas from cognitive science and reinforcement learning to begin address these questions.

# Background

- 1 Motivation
- 2 **Background**
  - Multiobjective optimization
  - Interactive methods
- 3 A new type of artificial decision maker
  - Artificial decision makers
  - Computational rationality
  - Preferences
  - Memory
- 4 Current state of research
- 5 Conclusions

For a general introduction, see<sup>1</sup>.

## A Multiobjective Optimization Problem

$$\min_{\mathbf{x} \in S} \mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_k(\mathbf{x})), \quad (1)$$

where  $\mathbf{x} = (x_1, x_2, \dots, x_n)^T$  is a *decision variable vector* consisting of  $n$  decision variables;  $S \subset \mathbb{R}^n$  is the *set of feasible decision variables*; and  $f_i(\mathbf{x}) : S \rightarrow \mathbb{R} (i = 1, \dots, k)$  are conflicting *objective functions* to be minimized.

---

<sup>1</sup>Kaisa Miettinen. *Nonlinear multiobjective optimization*. Boston: Kluwer Academic Publishers, 1999.



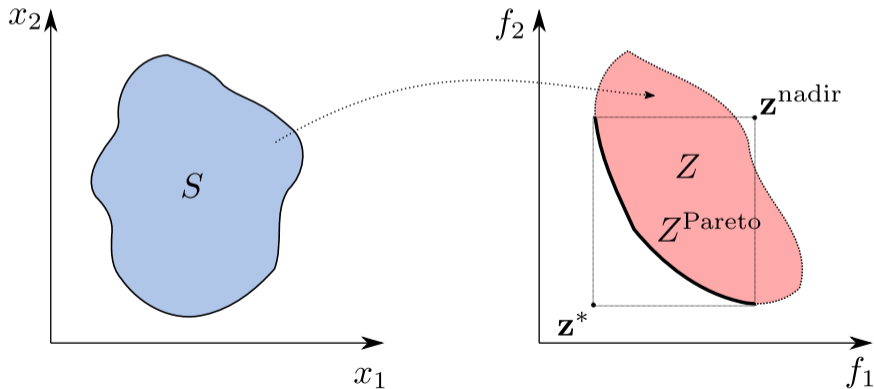
## Pareto optimality and the set of Pareto optimal solutions

If  $\mathbf{x}^*$  is a Pareto optimal solution to (1), then there does not exist another solution  $\mathbf{x}$  such that  $f_i(\mathbf{x}) \leq f_i(\mathbf{x}^*)$  for all  $i = 1, \dots, k$ , and  $f_i(\mathbf{x}) < f_i(\mathbf{x}^*)$  for some  $i = 1, \dots, k$ .

The set of all Pareto optimal solutions to (1) is known as the *set of Pareto optimal solutions*. The image of this set is  $Z^{\text{Pareto}}$ , and it is a subset of the image of the feasible set  $Z$ , i.e.,  $\mathbf{f} : S \rightarrow Z$  and  $Z^{\text{Pareto}} \subset Z$ .

# Multiobjective optimization III

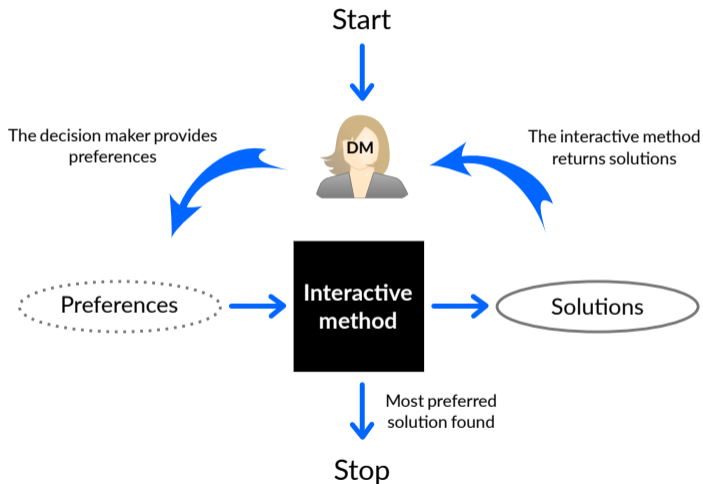
$$\min_{\mathbf{x} \in S} \mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}))$$



- Pareto optimal solutions cannot be fully compared.
- A decision maker (a domain expert) provides preferences.
- Multiobjective optimization methods utilize preferences to find the most preferred solution.

- Interactive methods: preferences provided iteratively during optimization.
- Preference type example: *reference point*  $\bar{\mathbf{z}} \in \mathbb{R}^k$  consisting of *aspiration levels*.
- Preferences change as the decision maker *constructs* (or *learns*) part of their preferences.

# Interactive methods III



# A new type of artificial decision maker

- 1 Motivation
- 2 Background
  - Multiobjective optimization
  - Interactive methods
- 3 A new type of artificial decision maker**
  - Artificial decision makers
  - Computational rationality
  - Preferences
  - Memory
- 4 Current state of research
- 5 Conclusions

- When decision makers are not available, how to study interactive methods?
- *Artificial decision makers* have been employed to compare interactive methods to some extent<sup>2</sup>.
- Do not capture human aspects, such as preference construction and cognitive limitations, such as memory.
- How to model interaction between a decision maker and an interactive method?

---

<sup>2</sup>Bekir Afsar, Kaisa Miettinen, and Francisco Ruiz. "Assessing the performance of interactive multiobjective optimization methods: A survey". 54.4 (2021). DOI: [10.1145/3448301](https://doi.org/10.1145/3448301).

# Computational rationality I

- *Computational rationality*: model human interaction by modeling its limitations<sup>3</sup>.
- Find *an optimal policy* utilizing a *reinforcement learning agent*<sup>4</sup> that describes optimal interaction under the limitations assumed.
- Interaction modeled as a *partially observable Markov decision process*.
- This approach has been utilized successfully in modeling driving a car<sup>5,6</sup> and typing on a touchscreen keyboard<sup>7</sup>, for example.

---

<sup>3</sup>Antti Oulasvirta, Jussi P. P. Jokinen, and Andrew Howes. "Computational Rationality as a Theory of Interaction". en. *CHI Conference on Human Factors in Computing Systems*. ACM, 2022, pp. 1–14. DOI: [10.1145/3491102.3517739](https://doi.org/10.1145/3491102.3517739).

<sup>4</sup>Richard S Sutton and Andrew G Barto. *Reinforcement learning: An introduction*. MIT press, 2018.

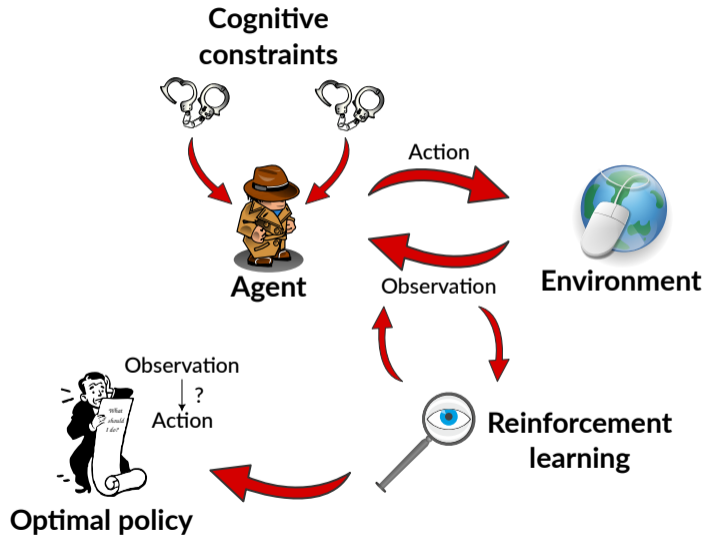
<sup>5</sup>Jussi P. P. Jokinen, Tuomo Kujala, and Antti Oulasvirta. "Multitasking in Driving as Optimal Adaptation Under Uncertainty". *Human Factors: The Journal of the Human Factors and Ergonomics Society* 63.8 (2021), pp. 1324–1341. DOI: [10.1177/0018720820927687](https://doi.org/10.1177/0018720820927687).

<sup>6</sup>Jussi P. P. Jokinen and Tuomo Kujala. "Modelling Drivers' Adaptation to Assistance Systems". *13th International Conference on Automotive User Interfaces and Interactive Vehicular Applications*. ACM, 2021, pp. 12–19. DOI: [10.1145/3409118.3475150](https://doi.org/10.1145/3409118.3475150).

<sup>7</sup>Jussi Jokinen, Aditya Acharya, Mohammad Uzair, Xinhui Jiang, and Antti Oulasvirta. "Touchscreen Typing As Optimal Supervisory Control". *Proceedings of the 2021 CHI Conference on Human Factors in Computing Systems*. ACM, 2021, pp. 1–14. DOI: [10.1145/3411764.3445483](https://doi.org/10.1145/3411764.3445483).



# Computational rationality II



- But when modeling interaction between a decision maker and an interactive method, what kind of limitations should we assume?
- For the purpose of this study, we have chosen two limiting factors: *preferences* and *memory*.



?

- Many theories exist on how preferences can be modeled in a decision maker.
- In our work, we have assumed that preferences are *constructed* during the decision process (the interactive process)<sup>8</sup>.
- Moreover, we have assumed that preferences are constructed from memory.

---

<sup>8</sup>Sarah Lichtenstein and Paul Slovic, eds. *The construction of preference*. Cambridge, England: Cambridge University Press, 2006.

- Decision makers prefer what is familiar to them.
- *Similarity*<sup>9</sup> between two objective vectors  $\mathbf{z}^1$  and  $\mathbf{z}^2$ :

## Similarity

$$\mathcal{S}(\mathbf{z}^1, \mathbf{z}^2) = \mathcal{S}^{(1,2)} = \exp \left( -\lambda \sum_{j=1}^k w_j |z_j^1 - z_j^2| \right), \quad (2)$$

where  $\lambda > 0$  is the *discriminability* of the solutions being compared, and  $0 \leq w_j \leq 1$  is the proportion of *attention* given to the  $j^{\text{th}}$  objective value. We assume that  $\sum_{j=1}^k w_j = 1$ .

<sup>9</sup>Jana B. Jarecki and Jörg Rieskamp. "Comparing attribute-based and memory-based preferential choice". *DECISION* 49.1 (2022), pp. 65–90.

- Similarity (2) is used to compute the *quality* of an objective vector  $\mathbf{z}$  by comparing it to memory:

### Quality based on similarity

$$\mathcal{V}(\mathbf{z}^i) = \mathcal{V}^i = \frac{\sum_e \mathcal{S}^{(i,e)} \times \mathcal{V}^e}{\sum_e \mathcal{S}^{(i,e)}}, \quad (3)$$

where  $e$  represents the other objective vectors, e.g., in memory, the objective vector  $\mathbf{z}^i$  is compared to, excluding the vector  $i$  itself.

- Quality: how much the decision maker prefers an objective vector (the higher, the better).

- But what about memory?
- Assume a *retrograde model* memory: old information is replaced by new one as it presents itself<sup>10,11</sup>.
- Memory is limited. Max  $m$  objective vectors in memory at any time.

---

<sup>10</sup>M. Katkov, S. Romani, and M. Tsodyks. "Memory Retrieval from First Principles". *Neuron* 94.5 (2017), pp. 1027–1032.

<sup>11</sup>Mikhail Katkov, Michelangelo Naim, Antonios Georgiou, and Misha Tsodyks. "Mathematical models of human memory". *Journal of Mathematical Physics* 63.7 (2022), p. 073303.

## Simple memory model

Suppose  $\mathcal{M}$  is a set of objective vectors such that  $|\mathcal{M}| < m$ , where  $m$  is the size of our memory. Then, a simple memory model based on the principle of retrograde memory and similarity looks like:

$$\begin{aligned} &\text{if } |\mathcal{M}| < m \text{ then } \mathcal{M} \leftarrow \mathcal{M} \cup \{\mathbf{z}^{\text{new}}\}, \\ &\text{else if } |\mathcal{M}| = m \text{ then } \mathcal{M} \leftarrow (\mathcal{M} \setminus \{\mathbf{z}^i\}) \cup \{\mathbf{z}^{\text{new}}\}, \\ &\quad \text{where } i = \operatorname{argmax}_{j \in [1, m]} [\mathcal{S}(\mathbf{z}^j, \mathbf{z}^{\text{new}})]. \end{aligned} \tag{4}$$

- When memory is not full, add new observation  $\mathbf{z}^{\text{new}}$  to it.
- When memory is full, replace most similar object with  $\mathbf{z}^{\text{new}}$ .

- Humans tend to “clump” similar things together in their memory.
- We can assume certain objective vectors to always stay in memory → steady part of preferences.
- Steady part can represent domain expertise.



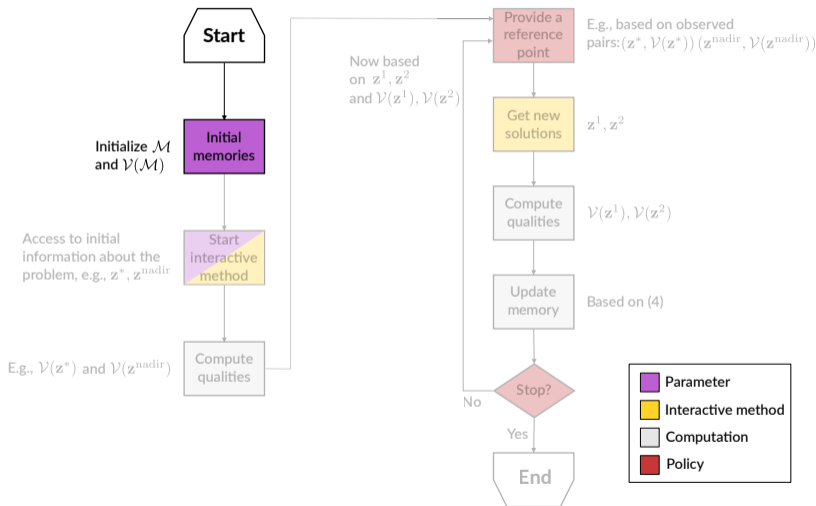
# Current state of research

- 1 Motivation
- 2 Background
  - Multiobjective optimization
  - Interactive methods
- 3 A new type of artificial decision maker
  - Artificial decision makers
  - Computational rationality
  - Preferences
  - Memory
- 4 **Current state of research**
- 5 Conclusions

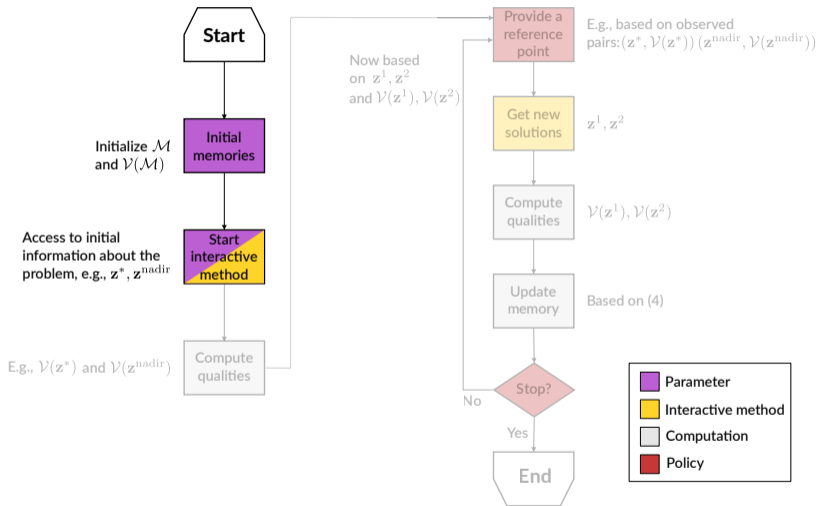
# What we are currently doing

- Reinforcement learning agent based on the preference (3) and memory models (4).
- Agent interacts with a reference point based interactive multiobjective optimization methods.
- Can work with other types of preferences.
- Experimenting with different reward functions.

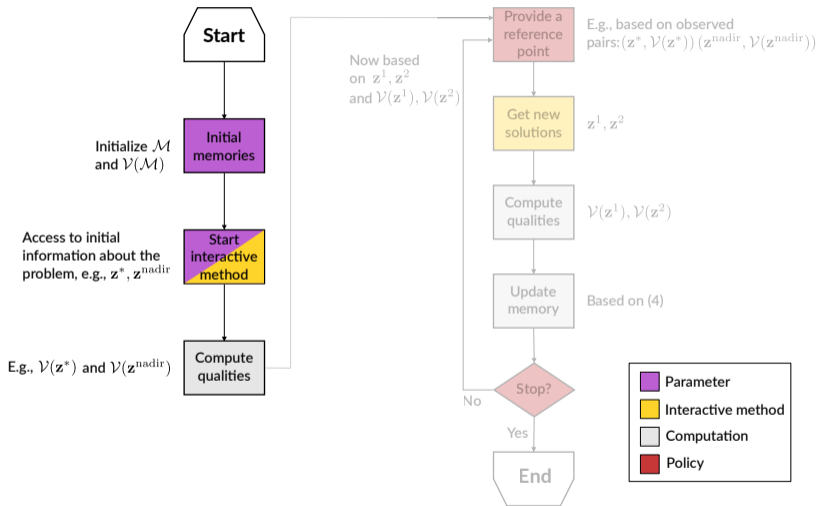
# Our reinforcement learning agent I



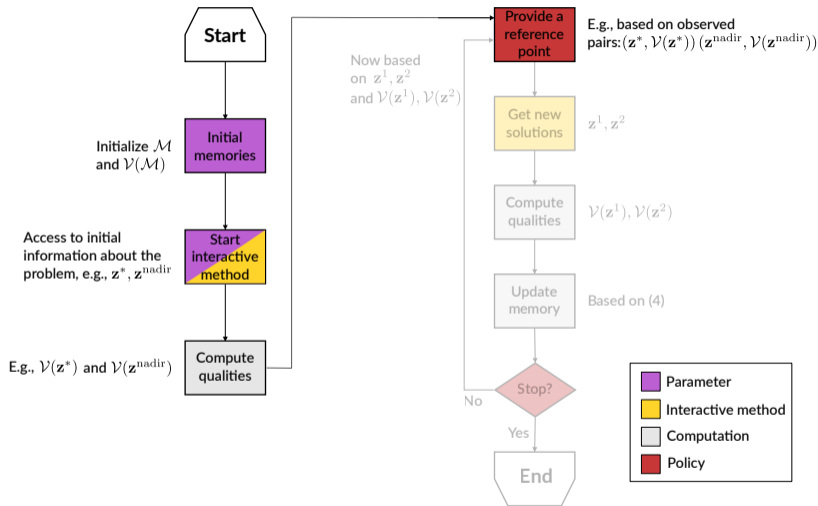
# Our reinforcement learning agent II



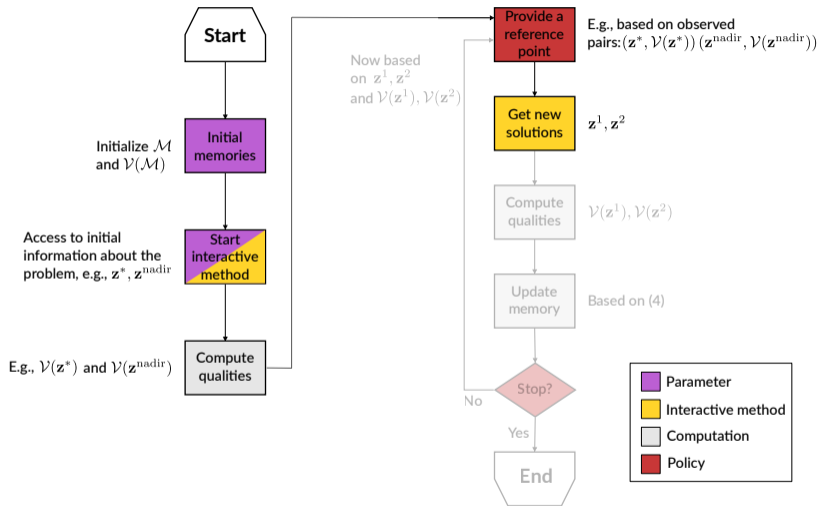
# Our reinforcement learning agent III



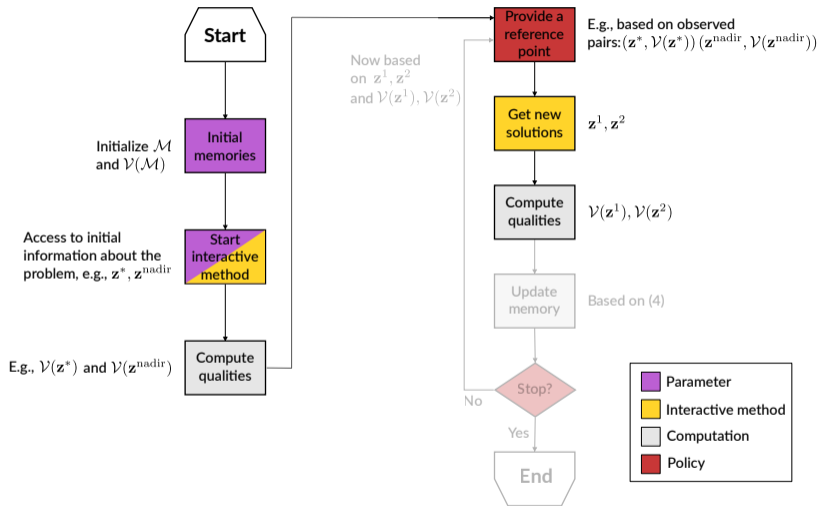
# Our reinforcement learning agent IV



# Our reinforcement learning agent $V$

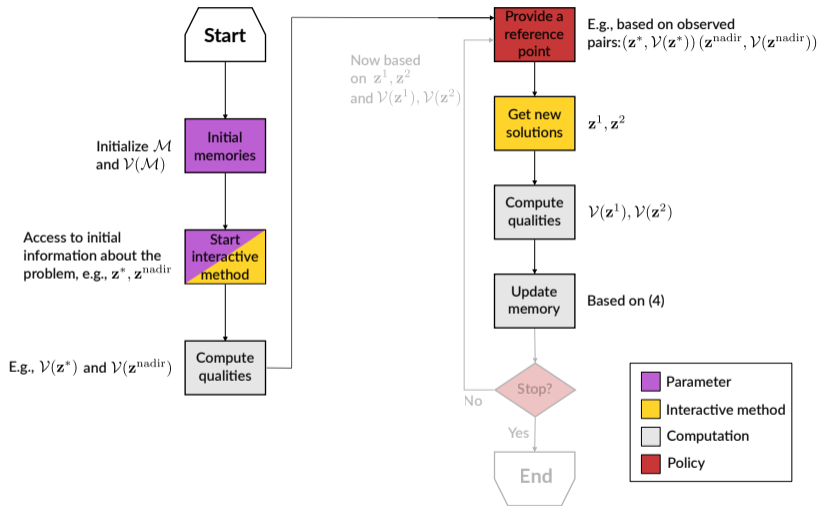


# Our reinforcement learning agent VI

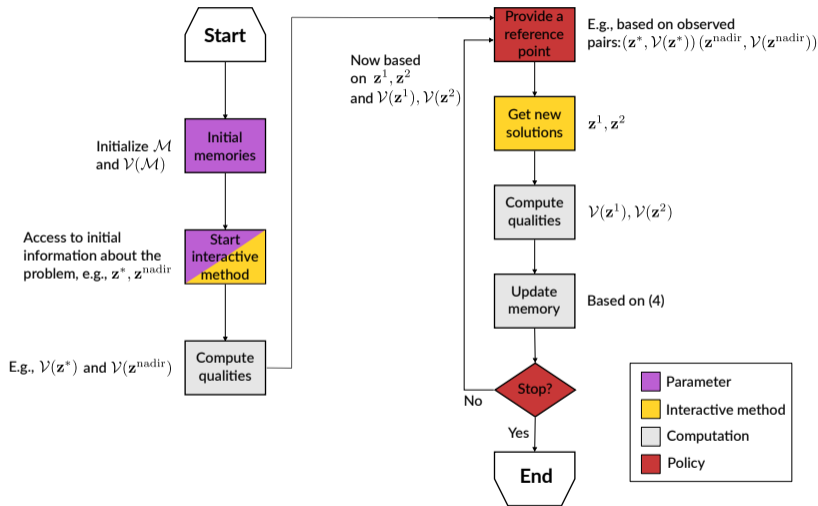




# Our reinforcement learning agent VII



# Our reinforcement learning agent VIII



# Our reinforcement learning agent IX

- Agent implemented in Python, gymnasium<sup>12</sup>, and stable-baselines3<sup>13</sup>.
- Finding optimal policies: Proximal Policy Approximation<sup>14</sup>.
- DESDEO<sup>15</sup> to model multiobjective optimization problems and to access interactive methods.
- Agent applied with the Reference Point Method<sup>16</sup>.

---

<sup>12</sup>Greg Brockman, Vicki Cheung, Ludwig Pettersson, Jonas Schneider, John Schulman, Jie Tang, and Wojciech Zaremba. "Openai gym". *arXiv preprint arXiv:1606.01540* (2016).

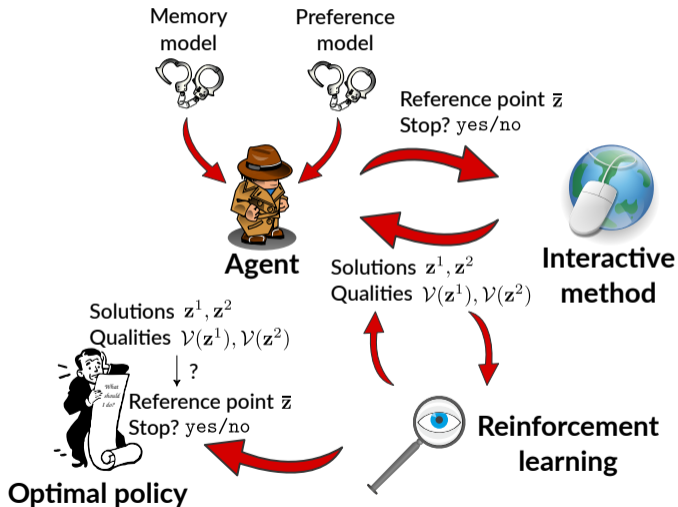
<sup>13</sup>Antonin Raffin, Ashley Hill, Adam Gleave, Anssi Kanervisto, Maximilian Ernestus, and Noah Dormann. "Stable-Baselines3: Reliable Reinforcement Learning Implementations". *Journal of Machine Learning Research* 22:268 (2021), pp. 1–8.

<sup>14</sup>John Schulman, Filip Wolski, Prafulla Dhariwal, Alec Radford, and Oleg Klimov. "Proximal policy optimization algorithms". *arXiv preprint arXiv:1707.06347* (2017).

<sup>15</sup>G. Misitano, B. S. Saini, B. Afsar, B. Shavazipour, and K. Miettinen. "DESDEO: The Modular and Open Source Framework for Interactive Multiobjective Optimization". *IEEE Access* 9 (2021), pp. 148277–148295.

<sup>16</sup>Andrzej P Wierzbicki. "A MATHEMATICAL BASIS FOR SATISFICING DECISION MAKING". *Mathematical Modelling* 3 (1982), p. 15.

# An optimal policy I



- By studying found policies, we can gather valuable information on how different interactive methods can help decision makers.
- Can policies be used to compare interactive methods?
- Can policies be used as a tools to assist decision makers during an interactive process?



## Remember these?

Two important questions arise:

- ① How can we compare interactive methods?
- ② How can we support different decision makers?

- Currently, the challenge is in finding a reward function that leads to a sensible policy. From such a policy, we can expect:
  - If an agent has initially an objective vector(s) in its memory that exists, or is very near, a Pareto optimal objective vector, then the agent should converge in very few iterations.
  - If an agent has initially an objective vector(s) in its memory that is not achievable and requires trading-off, we expect a longer deliberation before the agent decides to stop.
  - Ideally, we could be able to identify a *decision phase* and a *learning phase* in an agent following an optimal policy<sup>17</sup>.

---

<sup>17</sup>Jürgen Branke, Jürgen Branke, Kalyanmoy Deb, Kaisa Miettinen, and Roman Slowiński. *Multiobjective optimization: Interactive and evolutionary approaches*. Vol. 5252. Springer Science & Business Media, 2008.

- But how good is an optimal policy?
- We may only compare it to what we know from the literature when it comes to how decision makers may act during an interactive method.
- We need further studies to compare the policies to how real decision makers act.

# Conclusions

- 1 Motivation
- 2 Background
  - Multiobjective optimization
  - Interactive methods
- 3 A new type of artificial decision maker
  - Artificial decision makers
  - Computational rationality
  - Preferences
  - Memory
- 4 Current state of research
- 5 **Conclusions**



- New kind of artificial decision maker based on computational rationality and reinforcement learning.
- Limited by memory and preferences.
- Optimal policies can be useful for comparing interactive methods and to support decision makers.
- Studies comparing our artificial decision maker and real decision makers needed next.

# Let us network!

- My homepage: <http://giovanni.misitano.xyz>.
- Social media:
  - LinkedIn ([linkedin.com/in/misitano](https://www.linkedin.com/in/misitano)),
  - Twitter ([@misitano\\_g](https://twitter.com/misitano_g)), and
  - GitHub ([gialmisi](https://github.com/gialmisi)).
- DESDEO: <https://desdeo.it.jyu.fi/>
- The Multiobjective Optimization (research) Group:  
<http://www.mit.jyu.fi/optgroup/>



- [1] Kaisa Miettinen. *Nonlinear multiobjective optimization*. Boston: Kluwer Academic Publishers, 1999.
- [2] Bekir Afsar, Kaisa Miettinen, and Francisco Ruiz. “Assessing the performance of interactive multiobjective optimization methods: A survey”. *54.4* (2021). DOI: [10.1145/3448301](https://doi.org/10.1145/3448301).
- [3] Antti Oulasvirta, Jussi P. P. Jokinen, and Andrew Howes. “Computational Rationality as a Theory of Interaction”. en. *CHI Conference on Human Factors in Computing Systems*. ACM, 2022, pp. 1–14. DOI: [10.1145/3491102.3517739](https://doi.org/10.1145/3491102.3517739).
- [4] Richard S Sutton and Andrew G Barto. *Reinforcement learning: An introduction*. MIT press, 2018.

## Bibliography II

- [5] Jussi P. P. Jokinen, Tuomo Kujala, and Antti Oulasvirta. “Multitasking in Driving as Optimal Adaptation Under Uncertainty”. *Human Factors: The Journal of the Human Factors and Ergonomics Society* 63.8 (2021), pp. 1324–1341. DOI: [10.1177/0018720820927687](https://doi.org/10.1177/0018720820927687).
- [6] Jussi P. P. Jokinen and Tuomo Kujala. “Modelling Drivers’ Adaptation to Assistance Systems”. *13th International Conference on Automotive User Interfaces and Interactive Vehicular Applications*. ACM, 2021, pp. 12–19. DOI: [10.1145/3409118.3475150](https://doi.org/10.1145/3409118.3475150).
- [7] Jussi Jokinen, Aditya Acharya, Mohammad Uzair, Xinhui Jiang, and Antti Oulasvirta. “Touchscreen Typing As Optimal Supervisory Control”. *Proceedings of the 2021 CHI Conference on Human Factors in Computing Systems*. ACM, 2021, pp. 1–14. DOI: [10.1145/3411764.3445483](https://doi.org/10.1145/3411764.3445483).
- [8] Sarah Lichtenstein and Paul Slovic, eds. *The construction of preference*. Cambridge, England: Cambridge University Press, 2006.

## Bibliography III

- [9] Jana B. Jarecki and Jörg Rieskamp. “Comparing attribute-based and memory-based preferential choice”. *DECISION* 49.1 (2022), pp. 65–90.
- [10] M. Katkov, S. Romani, and M. Tsodyks. “Memory Retrieval from First Principles”. *Neuron* 94.5 (2017), pp. 1027–1032.
- [11] Mikhail Katkov, Michelangelo Naim, Antonios Georgiou, and Misha Tsodyks. “Mathematical models of human memory”. *Journal of Mathematical Physics* 63.7 (2022), p. 073303.
- [12] Greg Brockman, Vicki Cheung, Ludwig Pettersson, Jonas Schneider, John Schulman, Jie Tang, and Wojciech Zaremba. “Openai gym”. *arXiv preprint arXiv:1606.01540* (2016).
- [13] Antonin Raffin, Ashley Hill, Adam Gleave, Anssi Kanervisto, Maximilian Ernestus, and Noah Dormann. “Stable-Baselines3: Reliable Reinforcement Learning Implementations”. *Journal of Machine Learning Research* 22.268 (2021), pp. 1–8.

- [14] John Schulman, Filip Wolski, Prafulla Dhariwal, Alec Radford, and Oleg Klimov. “Proximal policy optimization algorithms”. *arXiv preprint arXiv:1707.06347* (2017).
- [15] G. Misitano, B. S. Saini, B. Afsar, B. Shavazipour, and K. Miettinen. “DESDEO: The Modular and Open Source Framework for Interactive Multiobjective Optimization”. *IEEE Access* 9 (2021), pp. 148277–148295.
- [16] Andrzej P Wierzbicki. “A MATHEMATICAL BASIS FOR SATISFICING DECISION MAKING”. *Mathematical Modelling* 3 (1982), p. 15.
- [17] Jurgen Branke, Jürgen Branke, Kalyanmoy Deb, Kaisa Miettinen, and Roman Slowiński. *Multiobjective optimization: Interactive and evolutionary approaches*. Vol. 5252. Springer Science & Business Media, 2008.